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Relay Feedback Identification for Processes Under Drift and Noisy Environments

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Relay feedback identification methods are widely used to find the process ultimate information and tune proportional-integral-derivative controllers. The conventional relay feedback method has several disadvantages, which include poor estimates of the process ultimate information for low-order processes, chattering of relay for noisy environments, and asymmetric relay responses for constant biases or slow drifts in the process outputs. Methods to mitigate each of the above disadvantages are available. However, a systematic method to treat all of them has not been studied yet. Here, simple relay feedback methods that resolve these problems by introducing band-pass filters in the feedback loop are proposed. The high-pass filter part in band-pass filter removes a constant bias or low frequency drift, and the low-pass filter part removes high frequency noise and high-order harmonic terms in the relay feedback oscillation, resulting better estimates of the process ultimate information. Because filters used for the proposed methods are able to reject constant biases, the process steady state gains can be estimated without disturbing the relay feedback oscillations and first order plus time delay (FOPTD) models can be obtained by combining the process steady state gains with the relay oscillation information. © 2010 American Institute of Chemical Engineers *AIChE J,* 57: 1809–1816, 2011
Keywords: relay feedback, drift, noise, ultimate data, filter, identification, FOPTD model

Introduction

A number of available proportional-integral-derivative (PID) controllers have autotuning features that are based on the relay feedback method. It is well known that a relay in the feedback loop produces a stable oscillation, which was first used by Astrom and Hagglund[1] to obtain process ultimate information and tune PID controllers. Now many variations and applications have been reported for autotuning of PID controllers.[2–4] They include methods to obtain more accurate ultimate information by suppressing the higher harmonic terms,[4–8] methods to obtain Nyquist points other than the critical point,[2,5,9–11] methods to reject unknown load disturbances and restore symmetric relay oscillations,[10–14] and methods to obtain the process steady state gain as well as the ultimate information.[15,16] Recently, Lee et al.[17] proposed identification methods to extract more accurate frequency response information and parametric models from a single conventional relay feedback test. They used various integrals of the original relay feedback responses to enhance identification performance without modifying the relay feedback system. Integrals of the relay responses make the

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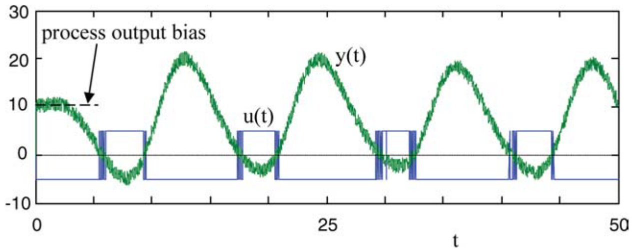

Figure 1. Responses of the conventional relay feedback system under drift and noisy environments (Process: $G(s) = 5/(s + 1)^6$), Sampling time = 0.02, Noise: uniform with a size of 3, Drift: 10 + 10sin(t/20)).

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fundamental frequency term dominant compared with the high harmonic terms, resulting in better accuracy in estimating frequency response information and model parameters.

For noisy environments, the original relay suffers from chattering. This chattering does not affect characteristics of relay oscillation such as stability. However, for some processes, chattering is not allowed. To avoid this chattering, a relay with hysteresis can be used. However, its switching period differs from the ultimate period.[2] A low-pass filter in the feedback loop may be used, but it also suffers from the same problem due to the phase lag of the filter. Recently dead-zone relay methods to provide offset-free ultimate data under noisy environments are investigated.[18] For processes with slow drifts or biases, relay feedback oscillations are asymmetric and result in errors for ultimate data estimation. Constant bias can be compensated by introducing input or output bias. The size of input or output bias is not known in advance and the compensating bias should be adjusted iteratively.[13] Methods to compensate for slow drifts are not available yet.

In this research, relay feedback methods to compensate for both high frequency noise and slow drifts are proposed. Band-pass filters are combined with relays such as the conventional relay and hysteresis relay. The high-pass part in band-pass filter treats slow drifts and makes the relay oscillation symmetric, and the low-pass part removes the effects of high frequency noise. Because band-pass filters can remove a slow drift, relay feedback methods with band-pass filters can be applied without waiting for processes to settle to their steady states. A filtered signal of the process output is available, and the proposed method utilizing this filtered signal will provide better estimates of ultimate information with reduced higher harmonic terms.

For some processes, information on the ultimate gain and period may not be enough to tune controllers, and it may be desirable to obtain additional process information that can be used to tune controllers. With one more piece of information in addition to ultimate gain and ultimate period, first order plus time delay (FOPTD) models are obtained by some authors.[3,15–17,19] A biased relay has been used to obtain the process steady state gain as well as the ultimate information[15] from only one relay feedback test. Huang et al.[16] used the integral of the relay transient to obtain the steady state gain of the process. Lee et al.[17] proposed identification methods to extract more accurate frequency response information and parametric models from a single conventional relay feedback test. In this article, a simple method that identifies the process steady state gain in addition to process frequency response data to fit a FOPTD model is proposed. Because the relay with band-pass filter removes a constant bias, a step change can be applied to the relay feedback loop without degrading the identification accuracy of frequency response, and a FOPTD model can be obtained.

Relay Feedback Methods Under Drifts and Noisy Environments

Consider a process

$$Y(s) = G(s)U(s) \tag{1}$$

where $U(s)$ and $Y(s)$ are Laplace transforms of the process input $u(t)$ and output $y(t)$, respectively, and $G(s)$ is the process transfer function. When a relay is put in the feedback loop, the closed loop will oscillate. Its oscillation period is near the ultimate period. The process ultimate gain can be obtained approximately by applying the describing function analysis to the oscillation amplitude. PID controllers can be tuned from the approximate ultimate period and gain.

The describing function analysis for the relay feedback response is based on ignoring higher harmonic terms. Hence the ultimate period and gain can have significant errors when the process $G(s)$ does not attenuate the higher harmonic terms in the square wave signal of the relay output sufficiently. There are many methods to reduce these errors by modifying relay types[4–8] or by applying different analyses.[17]

When the process output is corrupted with noise, the standard relay shows chattering (Figure 1), i.e., the relay switches quickly due to noise. To avoid this, a relay with hysteresis can be used, where the relay is switched on when $e(t)$ is greater than e_h and is switched off when $e(t)$ is less than $-e_h$ (e_h is the magnitude of relay hysteresis). Then the oscillation period is such that $G(j\omega) \approx -\frac{\pi}{4h}\left(\sqrt{a_y^2 - e_h^2} + je_h\right)$, where ω is the oscillation frequency, a_y is the amplitude of process output and h is the relay amplitude. The oscillation period is different from the ultimate period because $\angle G(j\omega) = -\pi + \arctan\left(e_h/\sqrt{a_y^2 - e_h^2}\right) \neq -\pi$. In addition, the noise makes it difficult to find the period and amplitude of oscillation accurately.

When slow drift is added to the process output, the relay oscillation is asymmetric (Figure 1). The ultimate data estimated from the asymmetric oscillation can have relative errors over 20%.[12] Iterative methods to restore symmetric responses for constant bias by compensating it with the input or output bias iteratively are available.[13] However, these iterative methods cannot be applied to processes with slow drifts.

Relay with Filters (RF)

Consider a conventional relay feedback system with filters (Figure 2)

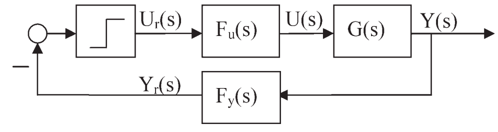

Figure 2. A relay feedback system with filters.

$$F_y(s) = F_L(s)F_H(s) = \frac{q\tau_F s + 1}{\tau_F s + 1}\frac{\tau_F s}{\tau_F s + 1} = \frac{\tau_F s(q\tau_F s + 1)}{(\tau_F s + 1)^2}$$

$$F_u(s) = F_y^{-1}(s) = \frac{(\tau_F s + 1)^2}{\tau_F s(q\tau_F s + 1)} \tag{2}$$

where $F_L(s) = (q\tau_F s+1)/(\tau_F s+1)$ and $F_H(s) = \tau_F s/(\tau_F s+1)$ are low-pass and high-pass filters, respectively. Because $F_u(s)G(s)F_y(s) = G(s)$ and consequently $Y_r(s) = G(s)U_r(s)$, both filters do not alter the oscillation characteristics of the relay feedback system except for the initial transient. The oscillation period of this relay feedback system with filters is the same as the conventional relay feedback system for the process $G(s)$. Hence, when the higher harmonic terms are attenuated sufficiently by $G(s)$, the process ultimate period P_u and the process ultimate gain K_u become[1,2]

$$P_u = p \qquad K_u = \frac{4h}{\pi a_r} \tag{3}$$

where p is the relay oscillation period, h is the relay amplitude, and a_r is the amplitude of filtered output $Y_r(s) = F_y(s)Y(s)$. In applying the relay feedback system with filters, a filtered signal

$$Y_{ri}(s) = \frac{1}{s}Y_r(s) = \frac{\tau_F(q\tau_F s + 1)}{(\tau_F s + 1)^2}Y(s) \tag{4}$$

is available. By analyzing this filtered output, a better estimate of the ultimate gain of process can be obtained. Since $Y_{ri}(s) \equiv \frac{1}{s}Y_r(s) = \frac{1}{s}F_y(s)G(s)F_u(s)U_r(s) = \frac{1}{s}G(s)U_r(s)$, the ultimate gain of the process can be replaced by

$$K_u = \frac{1}{|G(j\omega)|} = \frac{4h}{\pi a_{ri}}\left|\frac{1}{j\omega}\right| = \frac{2hp}{\pi^2 a_{ri}}, \quad \omega = \frac{2\pi}{p} \tag{5}$$

where a_{ri} is the oscillation amplitude of $Y_{ri}(s)$. The higher harmonic terms in $y_r(t)$ can be attenuated in $y_{ri}(t)$, and hence the estimate of Eq. 5 has lower error than that of the conventional relay feedback method.[17]

The low-pass filter part in $F_y(s)$, $F_L(s) = (q\tau_F s + 1)/(\tau_F s + 1)$, will remove high frequency noise when q is small. However, its inverse in $F_u(s)$, $F_L^{-1}(s) = (\tau_F s + 1)/(q\tau_F s + 1)$, will produce a sharp peak in the process input $u(t)$ for the relay switching. As q is made smaller, the peak is higher at the expense of deeper filtering effects. Here $q = 0.1$ is used.

The high-pass filter part in $F_y(s)$, $F_H(s) = \tau_F s/(\tau_F s + 1)$, will remove a constant bias and low frequency drift. The symmetric switching can be obtained even under these disturbances. This constant disturbance in the process is one of the obstacles in applying the relay feedback method. The conventional relay can even fail to produce a stable oscillation for certain large constant disturbances. A stable oscillation under a constant disturbance becomes asymmetric and provides a poor estimate of ultimate information. The high-pass filter relieves this disadvantage of the conventional relay feedback system. The inverse of the high-pass filter in $F_u(s)$ has integral action and can cause integral wind-up as in PID controllers. To avoid this wind-up, the integration is stopped when its value exceeds a given upper or lower limit.

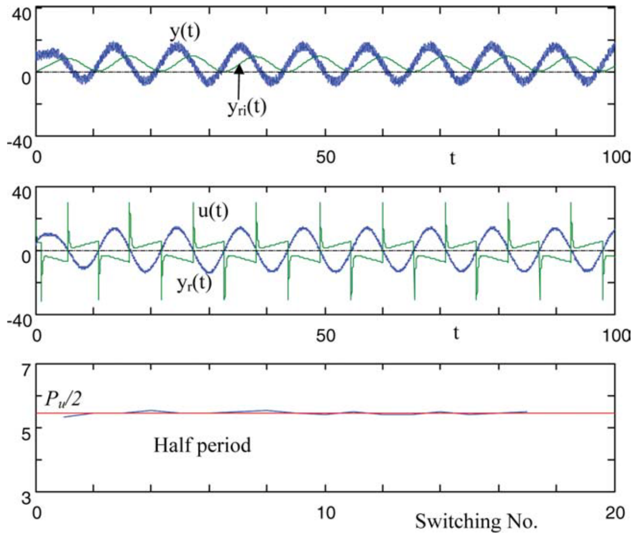

Figure 3. Relay with filters (RF) for slow drift and noise environment (Process: $G(s) = 5/(s+1)^6$, Sampling time = 0.02, Drift: 10 + 10sin(t/20), Filters: $F_y(s) = 2s(0.2s + 1)/(2s + 1)^2$ and $F_u(s) = (2s + 1)^2/(2s(0.2s + 1))$.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Figure 3 shows responses for the process $G(s) = 5/(s+1)^6$ with slow drift of $10 + 10\sin(t/20)$. The filter $F_y(s) = 2s(0.2s + 1)/(2s + 1)^2$ is used. It is seen that the bias and slow drift are eliminated in the filtered signal $y_r(t)$ and consequently symmetric relay switching is obtained. There is no significant offset in the ultimate data estimated.

A large filter time constant will make the initial transient be slow and more relay switchings will be required. However, the final oscillation characteristics are not affected by the filter time constant because the output filter is cancelled by the input filter. Hence it can be chosen so that drift and noise are attenuated sufficiently in the relay input, without worrying about identification performances. Figure 4 shows effects of filter time constants. It is seen that filter time constants different by 100 times provide relay oscillations which are very similar in period and amplitude.

Modified Dead-Zone Relay with Filters (MDRF)

Peaks in the process input of the previous RF method are due to jump changes of the relay and inverse of low-pass filter $F_L(s)$. This behavior may not be desirable for some applications. There are several chattering-free methods for noisy environments[2] and peaks in the process input can be removed because such methods do not need the low-pass filter $F_L^{-1}(s)$ to smooth noise. For this, a dead-zone relay whose dead-zone is determined by the integral of the process output[18] is considered for the identification of ultimate information without sharp peaks in the process input. The relay is switched as shown in Figure 5, where $y_r(t)$ is the filtered output and $y_{ri}(t)$ is its integral.

Step 1: The relay is off when $y_r(t)$ is positive.

Step 2: The relay is switched to zero when $y_r(t)$ crosses zero first. The value of $y_{ri}(t)$ is saved at that time. The relay output is kept at zero until $y_{ri}(t)$ crosses the saved value.

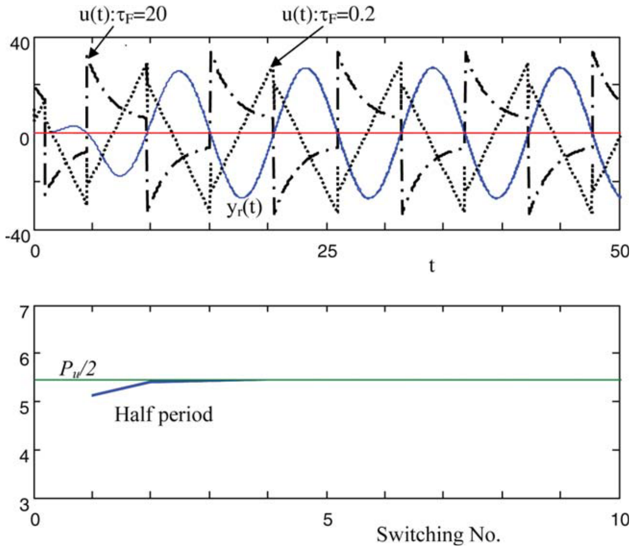

Figure 4. Effects of filter time constant for relay with filters (RF) (Process: $G(s) = 5/(s+1)^6$, Sampling time = 0.02, Drift: 0, Noise: 0).

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Step 3: The relay is switched on when $y_{ri}(t)$ crosses the saved value and is maintained for the negative process output.

Step 4: The relay is switched to zero when $y_r(t)$ crosses zero first. Save the value of $y_{ri}(t)$ at that time. The relay output is kept at zero until $y_{ri}(t)$ crosses the saved value.

Step 5: Repeat Steps 1 to 4 for a given number of oscillations.

Because this relay has the ability to avoid chattering for high frequency noise, a low-pass filter for $F_y(s)$ is not needed. Only a high-pass filter $F_y(s) = \tau_F s/(\tau_F s+1)$ to remove slow bias is used. The high-pass filter removes a constant bias and low frequency drift. Under these disturbances, a symmetric switching of the relay can be obtained. The input filter $F_u(s) = F_y(s)^{-1} = 1 + 1/(\tau_F s)$ has integral action and can cause integral wind-up. To avoid this, the integration is stopped when its value exceeds given upper or lower limit. The filter time constant can be chosen so that drift is attenuated sufficiently in the relay input.

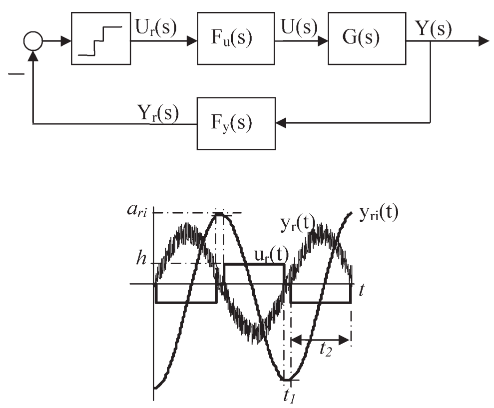

Figure 5. Modified dead-zone relay with filters.

Let t_1 and t_2 be durations of relay outputs of zero and on(off), respectively. Then the process ultimate period is $P_u = p = 2(t_1+t_2)$ and the process ultimate gain is[18]

$$K_u = \frac{2hp}{\pi^2 a_{ri}} \cos\left(\frac{t_1 \pi}{p}\right) \tag{6}$$

Figure 6 shows responses for the process $G(s) = 5/(s+1)^6$ with slow drift of $10 + 10\sin(t/20)$. Filters with $F_u(s) = 1 + 1/(2s)$ and $F_y(s) = 2s/(2s+1)$ are used. It is seen that the bias and slow drift are eliminated in the filtered signal $y_r(t)$, and consequently, the symmetric relay switching is obtained. There is no significant offset in the ultimate data estimated. This dead-zone relay can deal with high frequency noise without the low-pass filter.

When τ_F is large enough so that $\angle F_y(j\omega)$, the input filter $F_u(s)$ can be removed ($F_u(s) = 1$), and the process input is three-level. This is useful for some processes having input-nonlinearity.[8] However, very large τ_F causes long transients, requiring many relay switches. To avoid this, a second-order high-pass filter is considered;

$$F_y(s) = \frac{s(\tau_F s + 1)}{\tau_F^2 s^2 + \tau_F s + 1} \tag{7}$$

For $F_u(s) = 1$, $F_y(s)$ should satisfy

$$\begin{aligned} \angle F_y(j\omega) &= \frac{\pi}{2} + \arctan(\tau_F \omega) - \arctan\left(\frac{\tau_F \omega}{1 - \tau_F^2 \omega^2}\right) \\ &= \frac{\pi}{2} - \arctan\left((\tau_F \omega)^3\right) \approx 0 \end{aligned} \tag{8}$$

For that the phase error due to the filter $F_y(s)$ of Eq. 7 with $F_u(s)=1$ is less than δ, for a small δ,

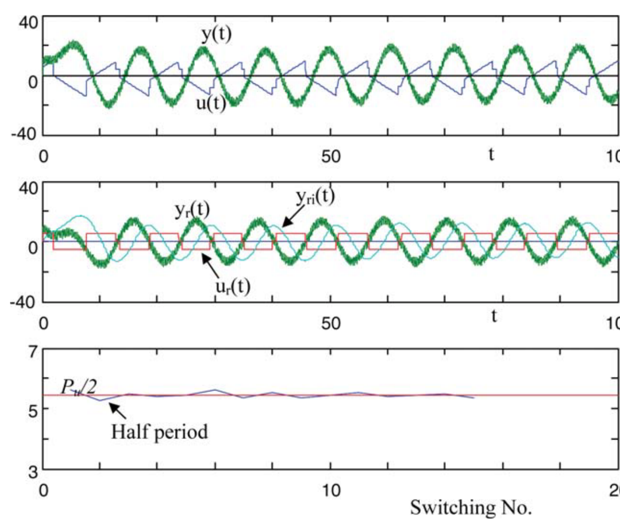

Figure 6. Modified dead-zone relay with filters (MDRF) for slow drift and noise environment (Process: $G(s) = 5/(s+1)^6$, Sampling time = 0.02, Noise: uniform with the size of 3, Drift: $10 + 10\sin(t/20)$, Filters: $F_u(s) = 1 + 1/(2s)$, $F_y(s) = 2s/(2s+1)$).

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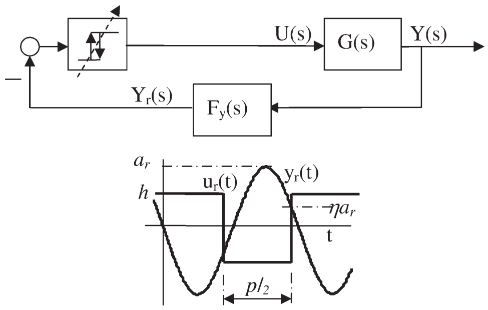

Figure 7. Adaptive hysteresis relay with a filter.

$$\tau_F\omega > 1/\tan(\delta)^{1/3} \approx 1/\delta^{0.333} \tag{9}$$

Specifically when $\tau_F\omega > 4$, the phase error is less than 1° and this condition can be used to specify τ_F.

Because the relay feedback oscillation is not known in advance, τ_F satisfying (9) with a sufficient margin is chosen in first two to three relay switchings. For the process $G(s) = 5/(s+1)^6$, filters of $F_u(s) = 1$ and $F_y(s) = s(10s + 1)/(100s^2 + 10s + 1)$ can be used. Since $P_u = 10.88$, the oscillation frequency is around $\omega = 0.58$ and the condition (9) is satisfied ($\tau_F\omega = 5.8 > 4$). The phase error due to the second-order high-pass filter, $F_y(s) = s(10s + 1)/(100s^2 + 10s + 1)$, will be less than 1°, and accurate estimation of the ultimate data can be obtained without $F_u(s)$.

Adaptive Hysteresis Relay with Filter (AHRF)

A hysteresis relay with a positive hysteresis provides phase lead, and this property is utilized here. Figure 7 shows the switching scheme of the hysteresis relay system. The relay is switching at ηa_r, where a_r is the amplitude of $y_r(t)$. Then the phase lead of the hysteresis relay is $\arcsin(\eta)$ and the input filter $F_u(s)$, which is introduced to compensate the phase lag of $F_y(s)$, can be removed. When the input filter ($F_u(s) = 1$) is removed, the on-off characteristics of process input is maintained, and the method can be applied to processes with input nonlinearities such as Hammerstein processes.[8]

Consider the hysteresis relay with filters of $F_u(s) = 1$ and

$$F_y(s) = \frac{s}{\tau_F^2 s^2 + 1.4\tau_F s + 1} \tag{10}$$

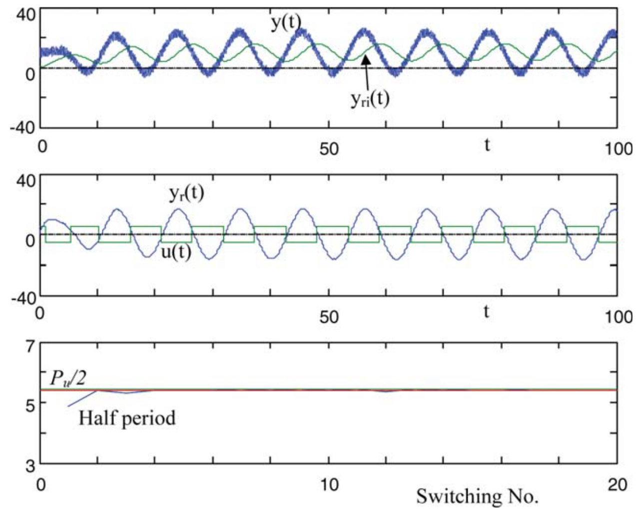

Figure 8. Adaptive hysteresis relay with filters (AHRF) for slow drift and noise environment (Process: $G(s) = 5/(s + 1)^6$), Sampling time = 0.02, Noise: uniform with the size of 3, Drift: 10 + 10sin(t/20), Filters: $F_u(s) = 1$, $F_y(s) = s/(4s^2 + 2.8s + 1)$.

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When the phase lag of the filter $F_y(s)$ and the phase lead of the hysteresis relay are in balance, i.e., $\arcsin(\eta) = -\angle F_y(j\omega)$ or

$$\eta = \sin\left(-\angle F_y(j\omega)\right) = \sin\left(-\pi/2 + \arctan\left(\frac{1.4\tau_F\omega}{(1-\tau_F^2\omega^2)}\right)\right) = \frac{-1+\tau_F^2\omega^2}{\sqrt{(1-\tau_F^2\omega^2)^2 + (1.4\tau_F\omega)^2}} \tag{11}$$

the oscillation period will be the ultimate period of process $G(s)$. The final relay feedback oscillation frequency ω is not known in advance and hence η should be adjusted iteratively at each relay switching point to satisfy Eq. 11. All simulations carried out for overdamped processes show that this iteration is convergent. The operation parameter η is needed to be near zero and, for this, the filter time constant should be such that $\angle F_y(j\omega) \approx 0$. The filter time constant may also be adjusted at the first two or three relay switching points when η is not near zero.

The process ultimate period is just the oscillation period and since the process input $u(t)$ is the square wave with period p and $Y_{ri}(s) \equiv \frac{1}{s}Y_r(s) = \frac{1}{s}F_y(s)G(s)U(s) = 1/[\tau_F^2 s^2 + 1.4\tau_F s + 1]G(s)U(s)$, the ultimate gain of the process can be replaced by

Table 1. Comparisons of Methods and Estimated Ultimate Periods

Method	Noise	Drift	Ultimate Period (P_u) Estimation	Ultimate Gain (K_{cu}) Estimation	Ultimate Period (P_u) for the process, $G(s) = 5/(s + 1)^6$ — Design Parameter	Noise	Noise and Drift
Classical relay	NA	NA					
Hysteresis relay	A	NA	W	W	$e_h = 1$	11.1 (2.9%)	
RF	A	A	S	B	$\tau_F = 2$		10.90 (0.2%)
MDRF	A	A	S	B	$\tau_F = 2$		10.87 (−0.1%)
AHRF	A	A	B	B	$\tau_F = 2$		10.87 (−0.1%)

NA: not applicable, A: applicable, W: worse, S: same, B: better.

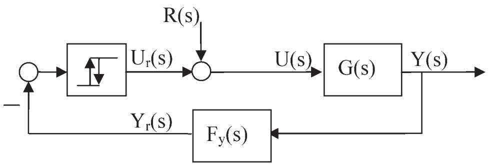

Figure 9. Hysteresis relay with a filter.

$$K_u = \frac{4h}{\pi a_{ri}} \frac{1}{\sqrt{(1-\tau_F^2\omega^2)^2 + (1.4\tau_F\omega)^2}}, \quad \omega = \frac{2\pi}{p} \tag{12}$$

It is remarked that, because the filter $F_y(s)$ reduces high-order harmonic terms further, the oscillation signal is closer to the sinusoidal wave than the previous relay feedback methods and better estimates of the process ultimate information can be obtained.

Figure 8 shows responses for the process $G(s) = 5/(s+1)^6$ with slow drift of $10 + 10\sin(t/20)$. Filters with $\tau_F = 2$ are used. It is seen that the bias and slow drift are eliminated in the filtered signal $y_r(t)$ and consequently the symmetric relay switching is obtained. There is no significant offset in the ultimate data estimated.

Combining various relays and filters, methods to identify ultimate gain and period of processes are proposed. Each method has its own advantages and disadvantages. Identification results for the process $G(s) = 5/(s+1)^6)$ are shown in Table 1. Here the sampling time is set to 0.02 and a uniform random noise with size of 1.5 and/or a drift of $10 + 10\sin(t/20)$ are added to the process output. For rather severe disturbances, identification of the ultimate period is excellent for all methods implemented.

FOPTD Model Identification

Many tuning rules based on the FOPTD model are available,[20] and they provide better tuning performance than those

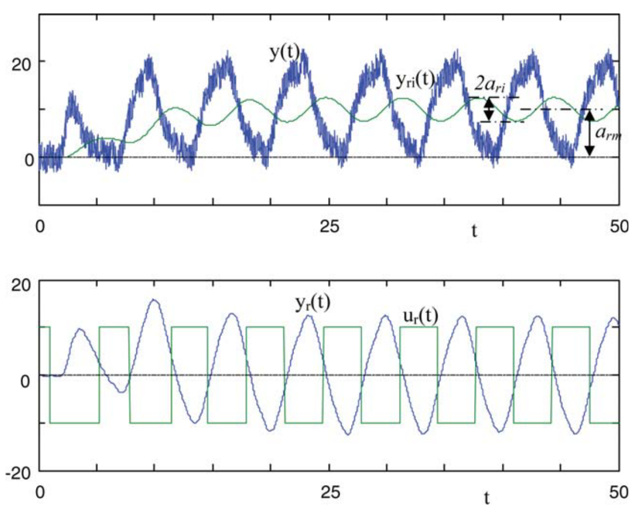

Figure 10. Hysteresis relay with a filter. (Process: *G*(*s*) = exp(−2*s*)/(*s* + 1)), Sampling time = 0.02, Noise: uniform with the size of 3, *r*(*t*): 10, Filter: $F_y(s) = s/(4s^2+2.8s+1)$.

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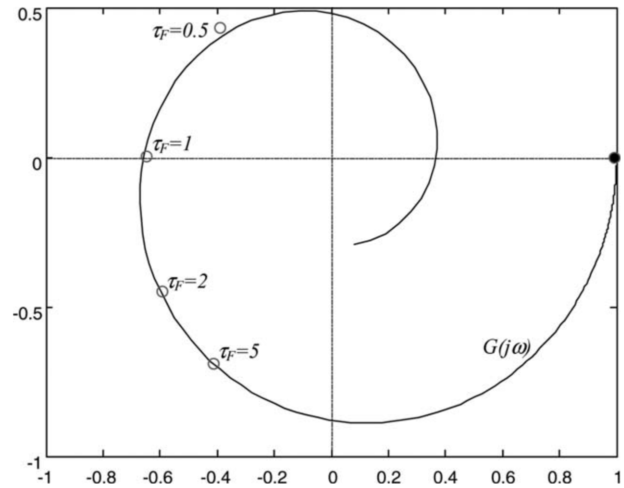

Figure 11. Nyquist plot of the process, *G*(*s*) = exp(−2*s*)/(*s* + 1), and identified points.

based on the process ultimate information only. The FOPTD model can be obtained from the process steady state gain in addition to the process ultimate information. The process steady state gain can be obtained by the biased relay method[15,21,22] or by analyzing the relay transient.[17] Here, utilizing the characteristics of high-pass filter to remove a constant disturbance, a simple method to find FOPTD model is proposed.

Consider a hysteresis relay system with the filter of Eq. 10 as shown in Figure 9 and its responses in Figure 10. Without adjusting η, a process frequency response at the phase angle

$$\phi = \angle G(j\omega) = -\frac{3\pi}{2} - \arctan\left(\frac{1.4\tau_F\omega}{1-\tau_F^2\omega^2}\right) - \arcsin(\eta) \tag{13}$$

can be identified. By changing the filter time constant τ_F and the hysteresis constant η, the identified Nyquist plot of process frequency response can be changed. To obtain the ultimate data, η or τ_F should be adjusted adaptively to be $\phi = -\pi$ as in the previous section. For the purpose of obtaining FOPTD model, it is not needed to adjust η iteratively. The amplitude ratio is

$$AR = |G(j\omega)| = \frac{\pi a_{ri}\sqrt{(1-\tau_F^2\omega^2)^2 + (1.4\tau_F\omega)^2}}{4h} \tag{14}$$

Figure 11 and Table 2 show Nyquist plots identified by changing τ_F for a fixed $\eta = 0.5$. Friman and Waller[9] suggested Nyquist point in the third quadrant to tune PID controllers. The optimal Nyquist point is dependent on process and controller types.

The band-pass filter $F_y(s)$ of Eq. 10 removes a slow drift and a constant bias. By applying a step change in $R(s)$ of Figure 9, the process steady state gain can be obtained as

$$G(0) = \frac{a_{rm}}{\Delta r} \tag{15}$$

where Δr is the size of step change in $R(s)$ and a_{rm} is the average value at the cyclic steady state of $y_{ri}(t)$ (Figure 10).

Table 2. Identification Results of the Proposed Hysteresis Relay with Filter for a First Order Plus Time Delay Process $G(s) = \exp(-2s)/(s + 1)$

		$G(j\omega)$ Identified			FOPTD Model Identified		
τ_F	p	$G(0)$	$\|G(j\omega)\|$ (Exact)	$\angle G(j\omega)$ (Exact)	k_c	τ	θ
0.5	4.32	1.0463	0.5804 (0.5666)	−3.9783 (−3.8774)	1.0463	0.9634	2.0497
1	5.50	0.9990	0.6493 (0.6587)	−3.1424 (−3.1366)	0.9990	0.9509	1.9676
2	6.86	0.9949	0.7413 (0.7374)	−2.4936 (−2.5734)	0.9949	1.0645	1.9087
5	8.32	0.9970	0.8043 (0.7980)	−2.1101 (−2.1572)	0.9970	1.0778	1.9107

Table 3. Identification Results of the Proposed Hysteresis Relay with Filter for Higher Order Plus Time Delay Processes

	Wang et al.[21]		Proposed Method		
Process	Model	IAE	τ_F	Model	IAE
$\frac{\exp(-2s)}{(2s+1)^2}$	$\frac{1.00\exp(-2.93s)}{4.072s+1}$	1.01	3	$\frac{1.0007\exp(-2.9598s)}{3.6776s+1}$	0.64
$\frac{(-s+1)\exp(-s)}{(s+1)^5}$	$\frac{1.00\exp(-4.24s)}{2.99s+1}$	0.57	5	$\frac{1.0027\exp(-4.7910s)}{2.2454s+1}$	0.35

From the process steady state gain of Eq. 15 and the frequency response of Eqs. 13 and 14, a FOPTD model as

$$\begin{aligned} G_m(s) &= \frac{k\ e^{-\theta s}}{\tau s+1} \\ k &= G(0) \\ \tau &= \frac{\sqrt{(k/AR)^2-1}}{\omega} \\ \theta &= \frac{-\phi - \arctan(\tau\omega)}{\omega} \end{aligned} \tag{16}$$

can be identified. Table 2 shows that accurate FOPTD models for FOPTD processes are obtained. Table 3 shows that the proposed method provides models with smaller integral of absolute error for the step responses. Figure 12 shows the Nyquist plots of two processes and their identified models.

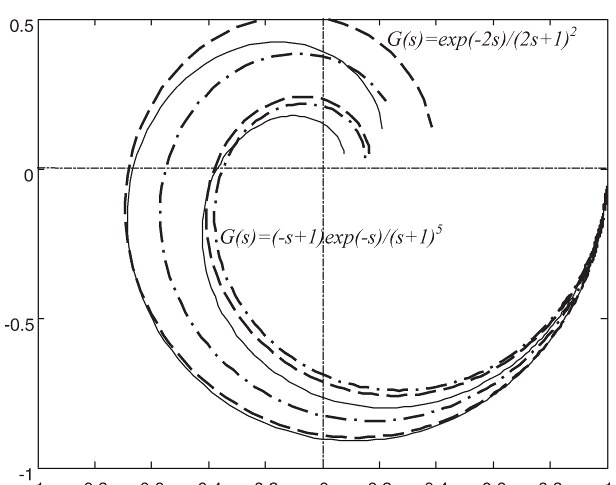

Figure 12. Nyquist plots of two processes and their identified FOPTD models (Solid line: process, Dashed line: proposed method, Dash-dotted line: Wang et al.[21]).

Conclusion

For noisy environments, a hysteresis relay is used for a chattering-free experiment to identify ultimate data for tuning of controllers. A smoothing scheme should also be applied to find the oscillation amplitude. Slow drifts make the relay feedback oscillations asymmetric and estimates of ultimate data will be poor. For a constant bias, iterative methods adjusting the input or output bias of relay have been used previously. In this article, methods that do not increase complexity and computational load much are proposed to identify ultimate data of processes under slow drift and noisy environments. Low-pass filters and high-pass filters are used to smooth high frequency noises and to remove effects of slow drifts including constant bias, respectively. Methods to compensate for phase shifts of those filters and obtain correct ultimate data of processes under slow drifts and high frequency noise are proposed.

High-pass filter removes a constant bias and hence a step change in the process input can be applied without disturbing the proposed relay feedback operations. This step change in the process input provides the process steady state gain. With the process steady state gain in addition to the process frequency response of relay feedback oscillation, a FOPTD model can be obtained. Simulations show that the proposed method to find FOPTD models is simple, and the models obtained are accurate for using controller tuning rules.

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Literature Cited

1. Astrom KJ, Hagglund T. Automatic tuning of simple regulators with specifications on phase and amplitude margins. *Automatica*. 1984; 20;645–651.
2. Astrom KJ, Hagglund T. *PID Controllers*. NC: Instrument Society of America, 1995.
3. Hang CC, Astrom KJ, Wang QG. Relay feedback auto-tuning of process controllers—a tutorial review. *J Process Control*. 2002;12; 143.
4. Yu CC. *Autotuning of PID Controllers: A Relay Feedback Approach*. London: Springer, 2006.
5. Tan KK., Lee TH, Huang S, Chua KY, Ferdous R. Improved critical point estimation using a preload relay. *J Process Control*. 2005; 16;445.
6. Sung SW, Park JH, Lee I. Modified relay feedback method. *Ind Eng Chem Res*. 1995;34;4133.
7. Sung SW, Lee I. Enhanced relay feedback method. *Ind Eng Chem Res*. 1997;36;5526.
8. Je CH, Lee J, Sung SW, Lee DH. Enhanced process activation method to remove harmonics and input nonlinearity. *J Process Control*. 2009;19;353–357.
9. Friman M, Waller KV. A two channel relay for autotuning. *Ind Eng Chem Res*. 1997;36;2662.
10. Sung SW, Lee J. A two-channel relay feedback method under static disturbances. *Ind Eng Chem Res*. 2006;45;4071.
11. Bi Q, Wang QG, Hang CC. Relay-based estimation of multiple points on process frequency response. *Automatica*. 1997;33;1753.
12. Shen SH, Wu JS, Yu CC. Autotune identification under load disturbance. *Ind Eng Chem Res*. 1996;35;1642.
13. Hang CC, Astrom KJ, Ho WK. Relay auto-tuning in the presence of static load disturbance. *Automatica*. 1993;29;563.
14. Sung SW, Lee J. Relay feedback method under large static disturbances. *Automatica*. 2006;42;353.
15. Shen SH, Wu JS, Yu CC. Use of biased relay feedback method for system identification. *AIChE J*. 1996;42;1174.
16. Huang HP, Jeng JC, Luo KY. Auto-tune system using single-run relay feedback test and model-based controller design. *J Process Control*. 2005;15;713.
17. Lee J, Sung SW, Edgar TF. Integrals of relay feedback responses for extracting process information. *AIChE J*. 2007;53; 2329–2338.
18. Kim JS, Byeon J, Chun D, Sung SW, Lee J, Edgar TF. Relay feedback method for processes under noisy environments. *AIChE J*. 2010;56;560–562.
19. Luyben WL. Getting more information from relay feedback tests. *Ind Eng Chem Res*. 2001;40;4391.
20. Seborg DE, Mellichamp DA, Edgar TF, Doyle FJ. *Process Dynamics and Control*, 3rd ed. NY: Wiley, 2010.
21. Wang QG, Hang CC, Zou B. Low-order modeling from relay feedback. *Ind Eng Chem Res*. 1997;36;375–381.
22. Panda RC, Yu CC. Shape factor of relay response curves and its use in autotuning. *J Process Control*. 2005;15;893.

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